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ABSTRACT ALGEBRA

Unit-1:Groups

Some basic properties of a Group

Theorem: In a group G, identity element is unique.

Proof: Let $(G, .)$ be a group.

Now we have to show that identity element in G is unique.

Suppose that $e_1, \ e_2$ be two identity elements in G.

For the identitys element e_1,

$$e_2 e_1 = e_1 e_2 = e_2, \text{ for } e_2 \in G \text{ -----(1)}$$

For the identity element e_2,

$$e_1 e_2 = e_2 e_1 = e_1 \text{ for } e_1 \in G \text{ -----(2)}$$

From (1) & (2), we have $e_1 = e_2$.

Therefore, we proved that identity element in a group is unique.

Theorem: In a group G, inverse of any element is unique.

Proof: Let $(G,\cdot)$ be a group and e be the identity element in G.

Let a be an arbitrary element in G.

To prove the theorem, we have to prove that inverse of a is unique.

Suppose that b and c be two inverse elements of a in G. Then

$$ab = ba = e \text{ and } ac = ca = e$$

Consider $c(ab) = c(e) = c$ ....(1) and

$$c(ab) = (ca)b = e.b = b \quad ....(2)$$

From (1)&(2), we have $b = c$.

Therefore the proof follows.

Theorem: Cancellation laws hold in a group. (or)

Let $\boldsymbol{G}$ be a group. Then for $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \boldsymbol{G}$,

$ab = ac \Rightarrow b = c$ (left) and

$ba = ca \Longrightarrow b = c$(right)

Proof: Let $(G, .)$ be a group.

Let e be the identity element in G.

For $a, b, c \in G$ and taking $ab = ac$.

Pre multiply the above equation with a^{-1} on both sides, we get

$$\Rightarrow a^{-1}(ab) = a^{-1}(ac)$$

$$\Rightarrow (a^{-1}a)b = (a^{-1}a)c$$

$$\Rightarrow eb = ec$$

$$\Rightarrow b = c.$$

Similarly, taking $ba = ca$

$\Rightarrow (ba)a^{-1} = (ca)a^{-1}$ (Post multiply with a^{-1})

$\Rightarrow b(aa^{-1}) = c(aa^{-1})$

$\Rightarrow be = ce$

$\Rightarrow b = c.$

Hence the Cancellation laws holds in a group.

Thank you